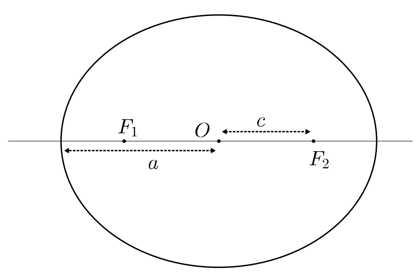
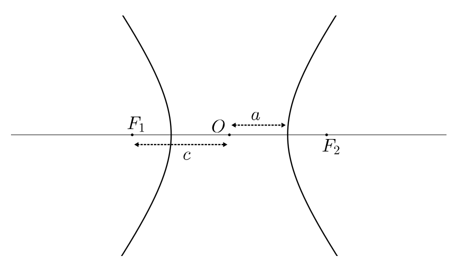
Conic Sections

17th January 2018

Def 1 An ellipse with foci F_1, F_2 is the locus of points P such that $F_1P + PF_2$ is constant. The major axis is the chord through the foci and a (the semimajor axis) is half its length. c (the linear eccentricity) is the distance between a focus and the midpoint of the major axis.

Def 2 A hyperbola with foci F_1, F_2 is the locus of points P such that $|F_1P - PF_2|$ is constant. The axis is the line through the foci and a is half the chord that lies on the axis. c (the linear eccentricity) is the distance between a focus and the midpoint of the major axis.





1. Prove that in an ellipse the ray from one focus reflects through the other focus.

2. Prove that for each focus of an elipse there exists a line (directrix) such that for any point on the ellipse the ratio of its distance from the focus to its distance from the directrix is constant. This ratio is called the eccentricity e. Find e in terms of a and c. What is the eccentricity of a circle?

3. Prove that a closed curve which is the intersection of a plane and a cone is an ellipse. (Hint: inscribe two spheres in the cone on either side of the plane - these are called Dandelin spheres)

4. Prove that for each focus of a hyperbola there exists a line (directrix) such that for any point on the hyperbola the ratio of its distance from the focus to its distance from the directirix is constant. This ratio is called the eccentricity e. Find e in terms of a and c.

5. Prove that a curve which is the intersection of a plane and both parts of a double cone is a hyperbola. (Hint: Dandelin spheres)

Def 3 A parabola is the locus of points equidistant from a focus and a directrix. Thus it has eccentricity 1.

6. Prove that all rays parallel to the axis of a parabola are reflected through the focus.

7. Prove that the curve which is the intersection of a cone and a plane parallel to its generating line is a parabola.

8. Prove that for a cone of semi-angle α, a plane at angle β to the axis of the cone cuts a conic section of eccentricity $\frac{\cos\beta}{\cos\alpha}$.

9. Find cartesian, polar and parametric equations for the conics in terms of a and e.