

Graph Theory

January 2017

1. Prove that a graph can not have an odd number of vertices of odd degree.
2. Prove that every graph has two vertices of equal degree.
3. Prove that a graph with all vertices of even degree can be partitioned into cycles.
4. Prove that a graph is bipartite if and only if it has no odd cycles.
5. Prove that if every vertex has degree of at least half the order of the graph, it is connected.

* Prove that it has a Hamiltonian cycle.
6. Prove that a connected graph that has one less edges than vertices is a tree [only use Euler's formula if you can prove it!].