Induction

September 2016

1. Prove that $n! > 2^n > n^2$ for $n \geq 4$. Generalise this to

$$\forall k, l \quad \exists m \quad \forall n > m \quad n! > k^n > n^l$$

2. Prove that if there are k lines on a plane in general position (no three intersect in one point, no two parallel), they divide the plane in $\frac{k(k+1)}{2} + 1$ parts.

3. Prove that $n^{n+1} > (n+1)^n$ for $n \geq 3$.

4. Prove that for any number of squares it is possible to divide them into pieces from which a larger square can be built.

5. Prove that the number 11...11 with 3^n 1s is divisible by 3^n.

6. Prove that for a real a, if $a + \frac{1}{a}$ is integer, than for any k, $a^k + \frac{1}{a^k}$ is integer.

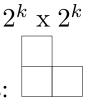
7. Prove that any whole number of pounds more than £7 can be paid with only £3 and £5 coins.

8. Prove that for any $n, m \quad (n+1)$ divides n^{2m-1}.

9. Prove that it is possible to number all sequences of 0s and 1s of a given length such that every sequence differs by exactly one term from the previous one.

10. Prove that the sum of the first n triangular numbers is $\frac{1}{6}n(n+1)(n+2)$.

11. Fermat's Little Theorem Prove that for a prime p, $a^{p-1} = 1\ (mod\ p)$.

12. One square has been removed from a 2^k x 2^k grid. Prove that it is possible to tile the rest with these 3-square corners: .