Invariants

November 2017

An invariant is something that doesn't change.

1. Glass A contains some water and glass B contains an equal volume of wine. Some of the contents of glass A is transferred to glass B, then the same amount of the resulting mixture in glass B is transferred to glass A. This is repeated 2017 times. Is there more water in glass B or wine in glass A?

2. A circle is divided into six segments and the numbers $1, 0, 1, 0, 0, 0$ are written in them in this order. Each turn you can add 1 to two adjacent sectors. Is it possible to make the numbers in all sectors equal? Find the necessary and sufficient conditions for the initial arrangement so that this is possible.

3. Each of $a_1, ..., a_n$ is either $+1$ or -1. It is given that

$$a_1a_2a_3a_4 + a_2a_3a_4a_5 + ... + a_na_1a_2a_3 = 0$$

Prove that $4|n$.

4. In a group of people, each person has at most three enemies. Is it possible to split the group into two parts so that each person has at most one enemy in the same part as them?

5. There are a azure, b blue and c cyan marbles. Each turn, two marbles of two different colours can be removed and replaced with a marble of the third colour. Is it possible to make all the marbles the same colour if initially $a = 13, b = 15, c = 17$? What are the necessary and sufficient conditions on a, b, c to make this possible?

6. Look up (or ask me about) Conway's soldiers.