

Inversion

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Inversion with centre O and radius r is a transformation that maps a point A to a point A' on the ray OA such that $OA \times OA' = r^2$

Prove the following properties of inversion:

1. $(A')'$ is A
2. A line through O inverts to itself.
3. $\triangle AOB$ is similar to $\triangle B'OA'$ for any A and B .
4. A line not through O inverts to a circle through O and conversely a circle through O to a line not through O .
5. A circle not through O inverts to another circle not through O .
6. * Tangency is conserved under inversion.
7. * The angle between two intersecting curves is conserved.

The cross-ratio of four points A, B, C, D is defined as

$$\frac{AC}{AD} \bigg/ \frac{BC}{BD}$$

8. Prove that the cross-ratio is conserved under inversion for four points on a line through O .
9. Prove that the cross-ratio is conserved for any four points.