

Plane Isometries

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Definition An isometry of the plane is a transformation that conserves distances between any two points.

1. How many point-image pairs define an isometry?
2. Prove that isometries conserve angles.
3. Prove that any isometry is the composition of at most 3 reflections.

Definition A glide reflection is the composition of a reflection and a translation parallel to the axis of reflection.

4. Prove that every isometry is either a translation, a rotation, a reflection, or a glide reflection.
5. Reconstruct an N -sided polygon, where N is odd, given the centres of its sides.
6. Consider points A of a shape S and their images under an isometry A' . Prove that the locus of the centre of AA' is either a point, a line, or a shape congruent to S .