# Pigeonhole Principle

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The principle states that if you put $n+1$ pigeons in $n$ pigeonholes, at least one pigeonhole will hold multiple pigeons.

*(I have since been informed that it is more common to put letters in pigeonholes rather than pigeons, but for the purpose of Dirichlet's principle they should be indistinguishable.)*

**1.** Can you generalise the principle? (What happens if you have more pigeons?)

**2.** $n$ people are in a room and some of them shake hands with each other. Prove that there are at least two people who have shaken hands the same number of times.

**3.** The integer lattice (all points in the xy-plane with integer coordinates) is coloured in red and blue in some way. Prove that there will always be a rectangle with vertices of only one colour.

**4.** Prove that given any 11 infinite decimals, at least two of them will have the same digit at infinitely many corresponding places.

**5.** 51, insects are placed inside a square of side 1. Prove that at any moment there at least three insects which can be covered by a single disk of radius $\frac{1}{7}$.

**6.** Any two people either know each other or are strangers. Prove that among 6 people, there are always 3 who either all know each other or are all strangers.

**7. Bézout's lemma** Let $a, b$ be coprime integers. Then there exist integers $x, y$ such that $ax + by = 1$.