

Similarity

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Definition Triangles ABC and $A'B'C'$ are said to be similar if $\angle ABC = \angle A'B'C'$, $\angle BCA = \angle B'C'A'$, $\angle CAB = \angle C'A'B'$.

Equivalently $\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CA}{C'A'}$.

1. Triangles ABC and $A'B'C'$ are similar with $AB = kA'B'$. Prove that $[ABC] = k^2[A'B'C']$.

2. Divide a given segment into n pieces.

3. (Bisector Theorem) Prove that $\frac{AB}{BD} = \frac{AC}{CD}$ if and only if AD is a bisector of $\triangle ABC$

4. The area of $\triangle ABC$ is S . Find the area of the triangle with the sides equal to the medians of $\triangle ABC$.

5. Prove that the polygon made by joining the vertices of a convex quadrilateral is always a parallelogram.

6. Given any $\triangle ABC$ prove that there exists $\triangle DEF$ similar to $\triangle ABC$ such that

$$[ABC] = 1(DE + EF + FD)$$

(1 is a unit of length, given for dimensional accuracy.)

7. Suppose that AB, CD are two parallel lines, E is the midpoint of CD , F and G the respective points of intersection of AC, BE and of AE, BD . Prove that FG is parallel to AB .

8. Through the vertex C of a parallelogram $ABCD$ a line is drawn meeting AB, AD in P, Q respectively. Prove that $BP \times DQ$ is constant.