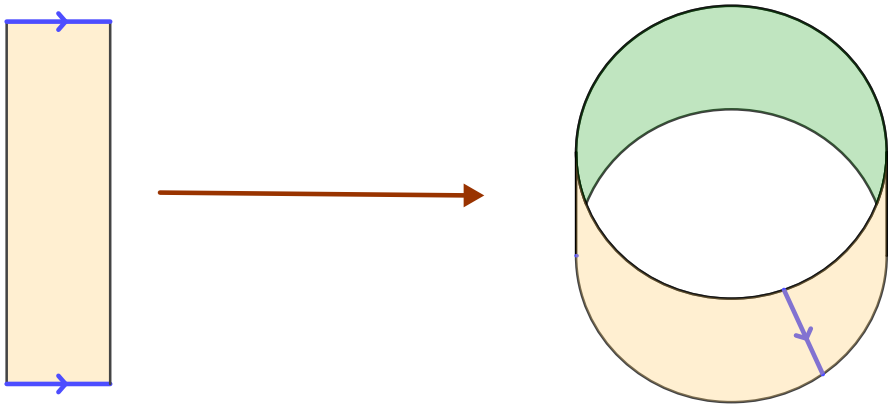
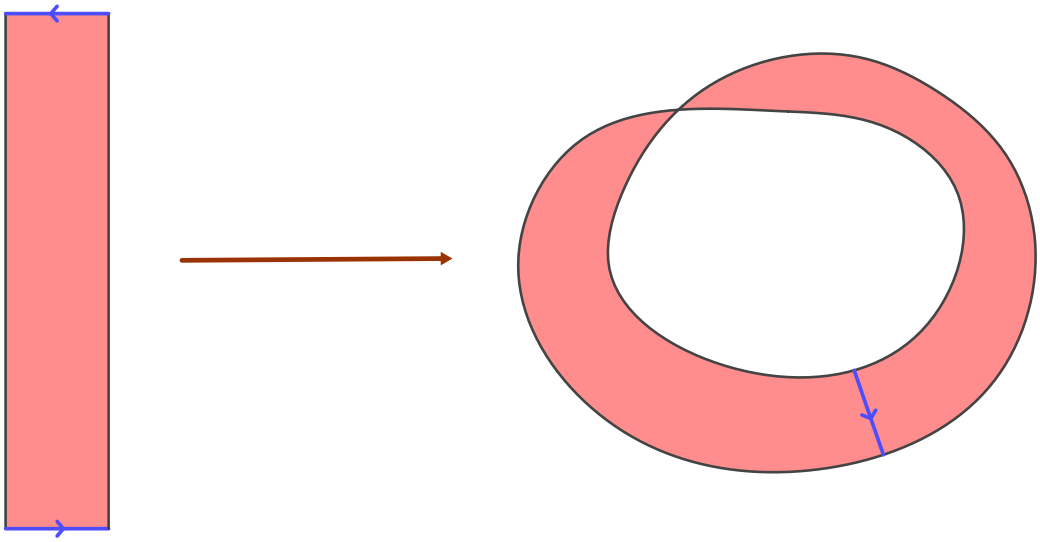
The Möbius Band

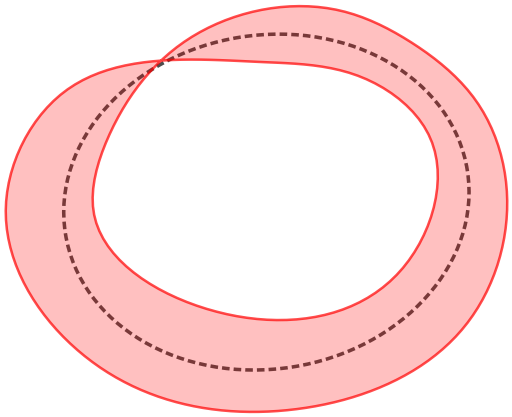
If you have a strip of paper, and you glue the ends together (along the blue lines in the picture), what do you get? It's a cylinder.



But what if, instead, you twist one of the ends when gluing? Then you get a remarkable shape called the Möbius band.



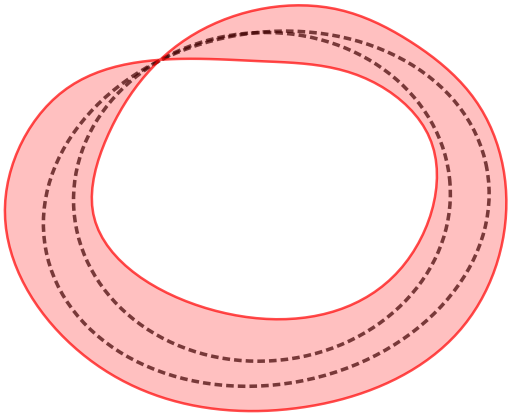
If you draw a line along the middle of the strip on a cylinder, you come back when you have gone a full circle, and one of the sides (the 'outside') now has a line on it. But you haven't drawn on the 'inside'. Now try with a Möbius band, what happens?



The line you draw goes over all of the band, there isn't any side of it left without the line! This is because the band's most important property is that it only has one side. When we put in the twist before gluing, we allowed ourselves to walk between what would have been the 'inside' and 'outside' on a cylinder, so now they are the same thing.

If you cut the cylinder along this middle line you drew, you get two cyliners. What if you cut along the middle line of the Möbius band? You only get a single piece of paper! This is because like it only has one side, the band also only has one edge, and we never cut it, so it couldn't have split into two pieces.

Now let's cut our shapes along the lines $\frac{1}{3}$ and $\frac{2}{3}$ along the strip. With the cylinder we'd just get three cylinders. What do you think we'll get with a Möbius band?



Now we get two connected loops, one twice as long as the other. The short one comes from the middle of the strip, and the long one is the two sides together - we still never cut the boundary.