Pascal's Triangle

Make a triangle of numbers: one number at the top, then two in the next row, three below and so on.
The top number and the ends of each row are 1s, for the others add up the two numbers above:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

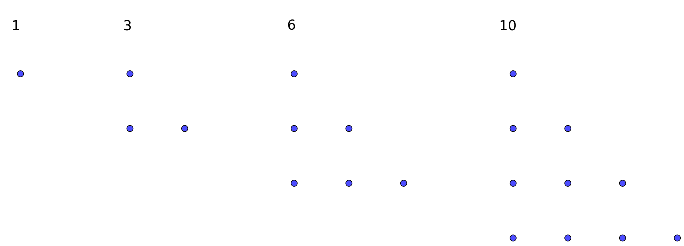
Patterns

Look at second number in every row: get (1,2,3,4,5,6...) - why?
Because we're just adding 1 each time!

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

Triangular Numbers

What do we get if we add up the first few whole numbers?
It's 1, then 1+2 = 3, then 1+2+3 = 6, 1+2+3+4 = 10 and so on. We can get these numbers by arranging dots into triangles, by ading new rows with one more dot each time:



This is why we call these the **triangular numbers**. And we can also find them in Pascal's triangle:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

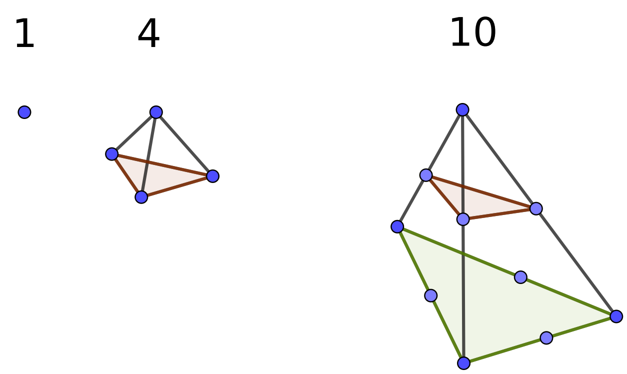
Why is that? If we call the first triangular number T_1, the second T_2 and so on (so $T_5 = 15$ for example), then we can see that $T_1 + 2 = T_2, T_2 + 3 = T_3$ and so on, so eg $T_5 + 6 = T_6$.
But then we're adding exactly these numbers in Pascal's triangle:

$$\begin{array}{ccccccccc}
 & & & & & & 1 & & \\
 & & & & & 1 & & 1 & \\
 & & & & 1 & & 2 & & T_1 \\
 & & & 1 & & 3 & & T_2 & \\
 & & 1 & & 4 & & T_3 & & \\
 & 1 & & 5 & & T_4 & & & \\
1 & & 6 & & T_5 & & & &
\end{array}$$

Pyramidal Numbers

We can also add up the first few triangular numbers to get the pyramidal (also called tetrahedral) numbers:
1, then 1+3 = 4, then 1+3+6 = 10, then 1+3+6+10 = 20 and so on...

You can think of these as stacking triangles in a pyramid:



Can you find the pyramidal numbers in Pascal's triangle? Think about why they show up!